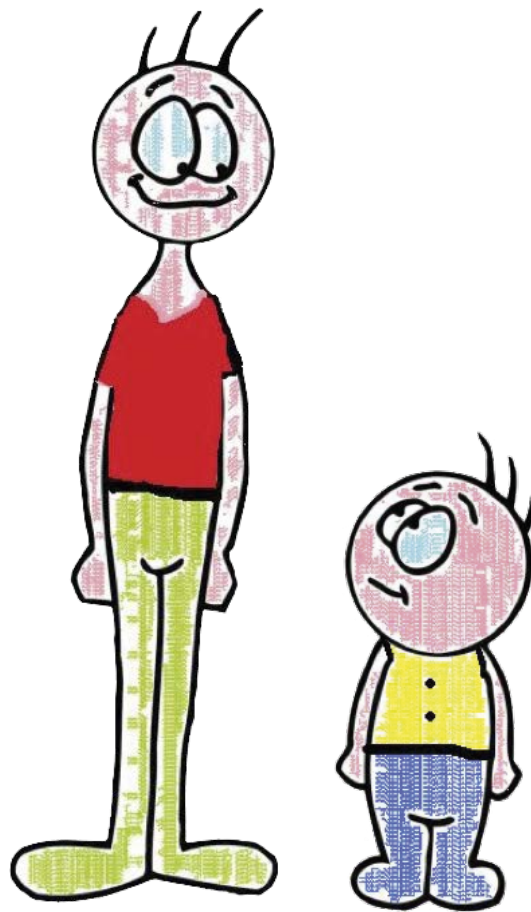# The long and the short of it
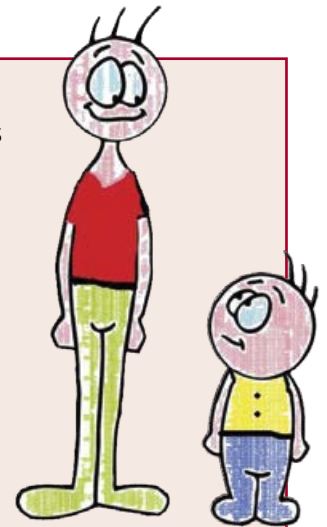
Support materials for teachers

Year 6 Reasoning in the classroom – The long and the short of it

These Year 6 activities start with an item that was included in the 2014 National Numeracy Tests (Reasoning). They continue with two linked activities, in which learners play different games.

Activity 1

The long and the short of it
Learners use their numerical skills to work out who will win a race.
Includes:
- Teachers' script
- PowerPoint presentation
- The long and the short of it questions
- Markscheme

Activity 2

The race
They interpret a graph of the race, completing a 'newspaper' report, then create a graph of their own race for others to interpret.
Includes:
- Explain and question – instructions for teachers
- Resource sheet – Mick and Sam's race
- Resource sheet – The race report

Activity 3

A different race
They play a game using positive and negative numbers, then create their own game.
Includes:
- Explain and question – instructions for teachers
- Whiteboard – Two-dice game
- Resource sheet – Our game board

Reasoning skills required

Identify	Communicate	Review
Learners choose their own methods, using a range of numerical skills.	They explain their approach, in writing and verbally.	They reflect on their findings, considering different outcomes.

Procedural skills

- **Multiplication and division**
- **Graphs**
- **Negative numbers**

Numerical language

- **Solution**
- **Graph**
- **Represent**
- **Horizontal/vertical**
- **x-axis, y-axis**
- **Coordinates**
- **Intersection**
- **Steeper**
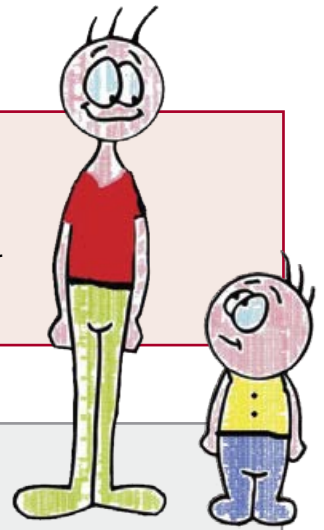- **Total**
- **Negative number**

The long and the short of it

Activity 1 – The long and the short of it

Outline

In this Year 6 activity, learners use their numerical skills to work out the positions of Mick and Sam during a race. They decide who has won the race, giving reasons for their decision.

You will need

Teachers' script

PowerPoint presentation
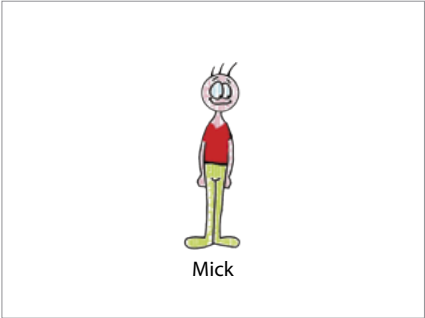
The long and the short of it questions
Two pages for each learner, must not be printed double-sided
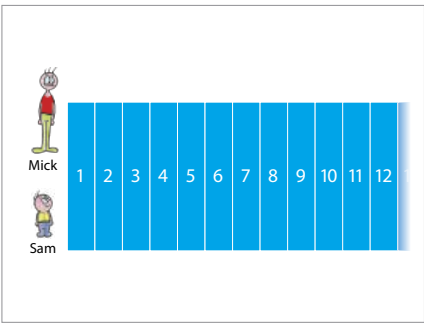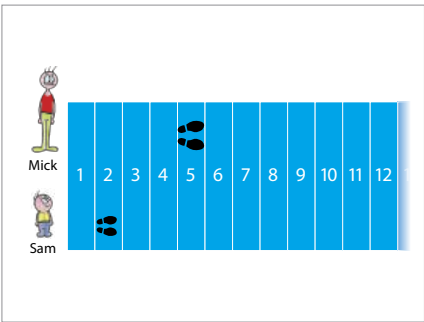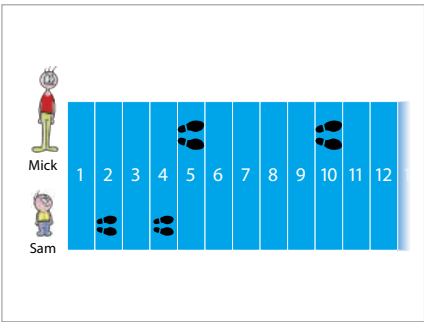
Markscheme

Presentation to be shown to learners before they work on The long and the short of it

The text in the right-hand boxes (but not italics) should be read to learners. You can use your own words, or provide additional explanation of contexts, if necessary. However, if you are using this as an assessment item, no help must be given with the numeracy that is to be assessed.

Slide 1		*(Keep this slide on the screen until you are ready to start the presentation.)*
Slide 2	Mick	This is Mick.
Slide 3	Sam	And this is Sam.

Slide 4		Here they are together, and you can see that Mick has much longer legs than Sam.
Slide 5		Mick and Sam are best friends. Their favourite game is jumping. They start together and then at the same time both boys do one jump.
Slide 6		The footprints show where they land. Mick lands on 5 but Sam's legs are much shorter than Mick's so he doesn't jump as far. Where does Sam land? That's right, on 2
Slide 7		Then, at the same time, they both do another jump. Now they have each jumped twice. Where is Mick? *(10)* When Mick is on 10, where is Sam? *(4)*

Slide 8		They jump again. Now they have each jumped three times. Where is Sam? *(6)*
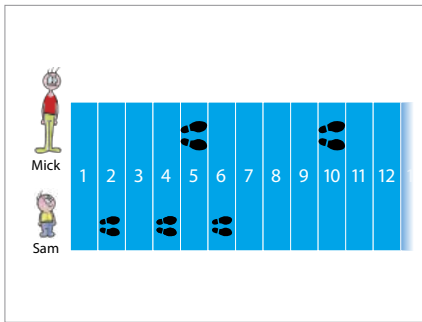
They jump again. Now they have each jumped three times. Where is Sam? *(6)*

When Sam is on 6, where is Mick? You can't see it, but can you work it out? Yes, he is on 15 because 10 add another 5 is 15

Tell me about Sam's jumps. How far does he jump each time? Yes, he jumps forward in twos like this . . . *(point to 2, 4, 6 but do not continue counting in twos beyond 6).*

Now tell me about Mick's jumps. How far does he jump each time? Yes, he jumps forward in fives like this . . . *(point to 5 and 10 and say 15, but do not continue counting in fives beyond 15).*

You are going to answer some questions about Mick and Sam and how they jump together.

Remember to show your working so that someone else can understand what you are doing and why.

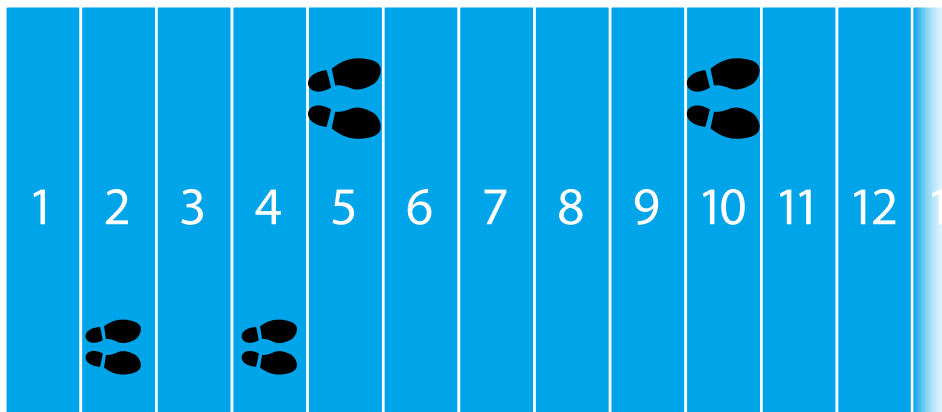*(If you are using this item for assessment purposes, you may wish to limit the time available, e.g. 10 minutes.)*
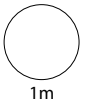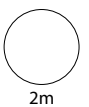
Mick

Sam

When **Mick** is on **20**, where is Sam?

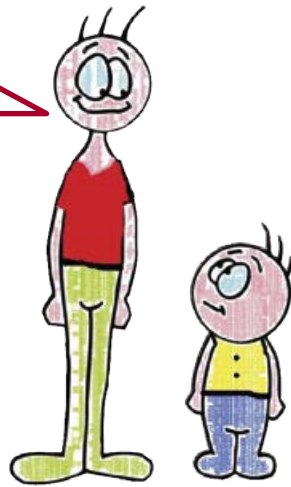When **Sam** is on **14**, where is Mick?

Mick and Sam start again.

They decide to race.

I will start on **0**
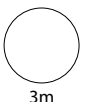
Your jumps are smaller so you start on **50**

Mick   **Sam**

Who gets to **100** first?

Show how you work it out.

Activity 1 – The long and the short of it – Markscheme

Q	Marks	Answer	
i	1m	**8**	

Q	Marks	Answer	
ii	2m	**35**	
	Or 1m	Shows the value **7**	◄ **Number of jumps**
		Or	
		Links **25** to **10** and **30** to **12**	◄ **Positions of Mick (M) and Sam (S) as they jump together**

Q	Marks	Answer	
iii	3m	M takes **20** jumps, S takes **25** jumps, so Mick wins	◄ **Number of jumps to get to 100**
		Or	
		Links M to **100**, S to **90**, so Mick wins	◄ **S's position when M reaches 100**
		Or	
		Links M to **125**, S to **100**, so Mick wins	◄ **M's position when S reaches 100**
	Or 2m	As for 3m, but gives a wrong or no conclusion	
		Or	
		Links M to **20** (accept 20 × 5)	◄ **Number of jumps to 100 for M**
		Or	
		Links S to **25** (accept 25 × 2)	◄ **Number of jumps to 100 for S**
	Or 1m	Makes a **list for S** that goes up in 2's, starting at 50 or 52 and ending at 100, even if there are errors	◄ **Finding number of jumps for S**
		Or	
		Makes a **list for M** that goes up in 5's, starting at 0 or 5 and ending at 100, even if there are errors	◄ **Finding number of jumps for M**
		Or	
		Links 5 to 52, 10 to 54, 15 to 56, and 20 to 58	◄ **Positions of M and S as they jump together**

Activity 1 – The long and the short of it – Exemplars

Part ii

When **Sam** is on **14**, where is Mick?	
2 4 6 8 10 12 5 10 15 20 25 30 30	The pairs 10, 25 and 12, 30 are shown; **1 mark** • The learner has continued the number patterns but stopped, perhaps because the diagram extends only as far as 12

Part iii

Who gets to **100** first? *Mick* Show how you work it out. Sam ____ Mick 50 ____ 0 ∨ ____ ∨ 25 spaces ____ 20	Shows that M takes 20 jumps, S takes 25 jumps and M wins; **3 marks**
Who gets to **100** first? Show how you work it out. Sam 50, 52, 54, 56, 58, 60 62, 64, 66, 68, 70 72, 74, 76, 78, 80 82, 84, 86, 88, 90 92, 94, 96, 98, 100 Mick 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70 75, 80, 85, 90, 95, 100 Sam 25 jumps Mick 20 jumps	Links M to 20 and S to 25 but there is no conclusion; **2 marks** • Although repeated addition is a correct method, it is inefficient. This learner may not be confident with multiplication and division.
5 52 10 54 15 56 20 58 25 60 30 62 35 64 40 66 45 68 50 70 55 72 60 74 65 76 70 78 75 80 80 82 85 84 90 88 95 92 Mick does get to 100 first	Shows 5, 52 then 10, 54 then 15, 56 then 20, 58; **1 mark** • This method would lead to a correct solution but, perhaps because the pairs of numbers are not written systematically, the learner has become confused.

The race

Activity 2 – The race

Outline

This activity continues the theme of a race as introduced in **Activity 1 – The long and the short of it**. Learners use a graph of the race to complete a report and solve a related problem. Then they create a graph of their own race and write a report, leaving elements of it for other groups to complete.
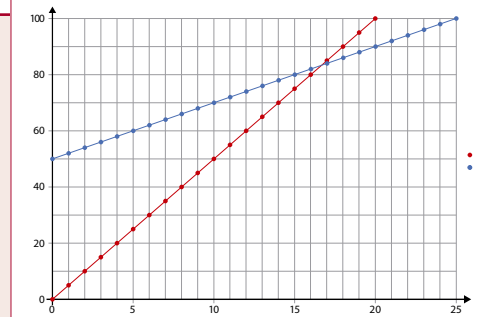
You will need

R   **Resource sheet – Mick and Sam's race**

R   **Resource sheet – The race report**

   Squared or graph paper

Activity 2 – The race

Explain

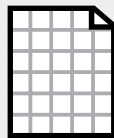Give each group/pair a copy of the resource sheet **Mick and Sam's race** and also a copy of the resource sheet **The race report**. Remind learners of Mick and Sam from **Activity 1 – The long and the short of it**, and how they jump together, then ask them to use the graph to complete the report.
(Solutions, in order: 0, 5; 50, 2; 30, 62; 10, 50; 17th; 20; 25)

Bring the class together to discuss the solutions, then say that as the boys are good friends, in their next race they would like to finish together. If Mick starts at 0, where should Sam start? Ask learners to work in their groups/pairs to find the solution and draw the graph of the race. *(This could be drawn on the graph on the resource sheet.)*

(As Mick takes 20 jumps to get from 0 to 100, Sam must also take 20 jumps. Each of his jumps is 2 squares, so he must travel 20 × 2 = 40, therefore he starts at 60.)

Finally, ask learners to think of two *(or more)* fictional characters of their own and choose each person's jump size and starting point. They should create a graph to show the race, then a report, with blanks, for other groups to complete.

Question

- On the graph, only some of the numbers on the axes are shown. Why don't we show them all? *(Not enough room)* What does each vertical line represent? *(One jump)* What does each horizontal line represent? *(10 squares jumped)*

- What do the points on the graph represent? *(The position of each boy)* What do the lines joining the points represent? *(Nothing, they are there to help the eye connect the points)*

- When we read a point on the graph, which value do we read first? *(The x-axis, i.e. the horizontal value)* So what are the coordinates of this point?

- What does this point *(the intersection of the two lines)* represent? *(Where the two boys meet)*

- Who is in the lead for most of the race? *(Sam)* How do you know when Mick overtakes him? *(Mick's points are higher up the y-axis.)*

- Whose line is steeper? *(Mick's)* Why? *(His jumps are bigger so he moves more each time.)*

- *(When the race is a draw)* How are you going to start this problem? What do you know already?

- *(When creating their own race)* Who is racing? What jumps are they taking? Where are they starting? Who do you want to win?

- Have you checked your report? How? Is all the information needed available on your graph? Have you kept a record of the solutions? Which of your questions is easiest/most difficult? Why?

100

80

Which
square
they
are on

60

40

20

0

0   5   10   15   20   25

Number of jumps

Mick   Sam
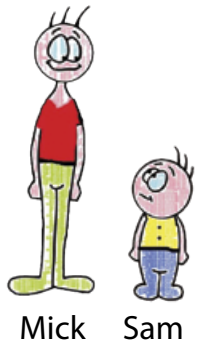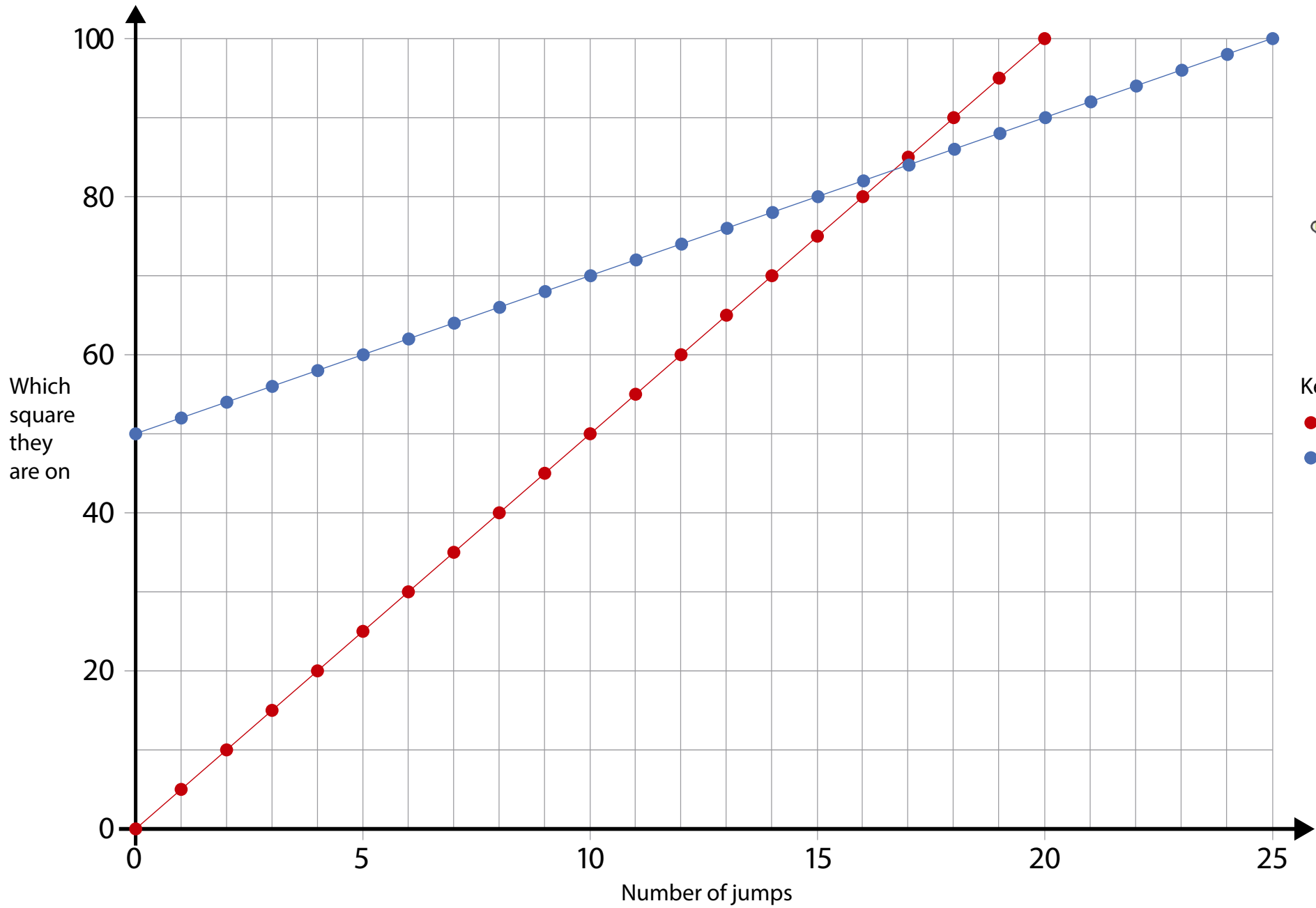
Key:

● Mick's jumps
● Sam's jumps

R

Race takes place!

The long-awaited race between Mick and Sam took place today. Crowds of people gathered to watch the race – everyone was tense. Who would win?

Mick started at _____ and jumped _____ squares at a time.

Sam started at _____ and jumped _____ squares at a time.

After 6 jumps, Mick was on _____ and Sam was on _____ .
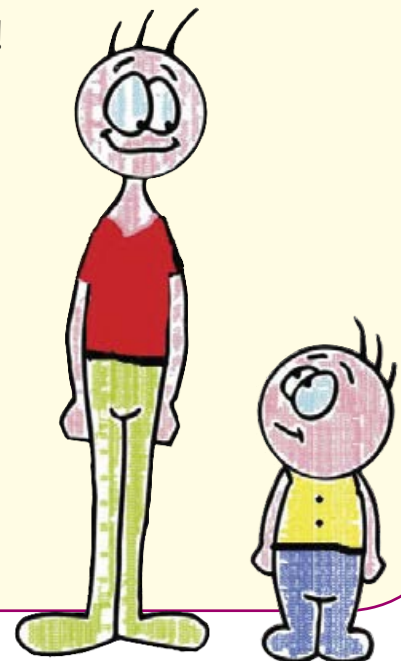
After _____ jumps, Mick was on _____ and Sam was on 70.

The gap was narrowing!

Would Mick catch Sam? Yes . . . by the _____th jump Mick was ahead of Sam for the first time and after _____ jumps he reached 100 which is the end of the race. Well done Mick!

And well done to Sam too, who reached 100 after _____ jumps.

We look forward to their next race!

Activity 3

A different race

Activity 3 – A different race

Outline

The theme of racing is continued in this activity, which focuses on a race using positive and negative numbers. As part of the activity is for learners to construct their own number sentences, you may wish to work with a small group at a time or have additional adult support.
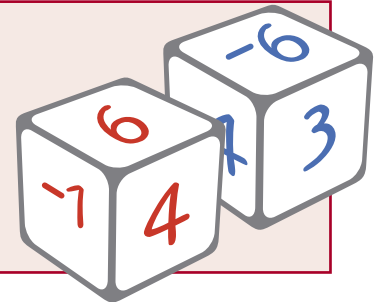
You will need

WB   **Whiteboard – Two-dice game**

R   **Resource sheet – Our game board**
One sheet per group/pair

Each group of two pairs needs two blank dice (widely available on the internet) which you will need to number as shown on the whiteboard two-dice game

Glue/tape for sticking the strips of board together

Card for making their own game

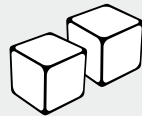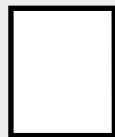# Activity 3 – A different race

(Note: throughout, 'negative' describes a number below zero, e.g. ⁻3 is read as 'negative 3'. This differentiates it from the operation which can be read as 'subtract' or 'minus'. So '4 + ⁻3' is read as '4 add negative 3' and '⁻3 + 2' is read as 'negative 3 add 2'. Although it is not used here, '⁻4 – ⁻3 = ⁻1' would be read as 'negative 4 minus (or subtract) negative 3 equals negative 1'.)

Show **Two-dice game** on the whiteboard and say this is a game for two pairs of players. It uses two dice, as shown. Each pair will start with a counter on 0. They throw both dice and find the total, then move their counter that number of places along the board. They take it in turns until one of them goes beyond 10 or ⁻10, in which case they are out and the other pair wins. Demonstrate on the whiteboard.

Once learners understand the rules, say that they are going to play the game – but first they need to create their own game board that goes from ⁻20 to 20. Give each group of two pairs a copy of the resource sheet **Our game board**, which they cut out and glue to make a long strip. Let them play the game, then use the questions below to probe their understanding.

Finally, learners can make their own game using negative numbers for other groups to play. *(This could involve spinners if no more blank dice are available.)*

Question

- Tell me what a negative number is. When do we use them in real life? *(For example, temperatures)*

- What is the greatest number of moves forward you could make? *(11, from 5 + 6)* What about the greatest number of moves backwards? *(⁻11, from ⁻5 and ⁻6)*

- What is the smallest number of throws you could have before you are out of the game? *(2)* What about the greatest number of throws? *(In theory, infinite)*

- Suppose someone throws the dice and doesn't move. What numbers could they have thrown? *(1 and ⁻1, 2 and ⁻2, etc.)*

- What if they move from 3 to 7? *(6 and ⁻2, 5 and ⁻1)* Or 7 to 3? *(⁻6 and 2, ⁻5 and 1)*

- Can you show me all possible moves? *(A table, as shown right, is an efficient method.)* Which is more likely – that you move forwards, backwards or stay still? *(Moving forwards or backwards are equally likely, and both are more likely than staying still. If appropriate, learners can use probabilities to describe outcomes.)*

+	⁻1	2	⁻3	4	⁻5	6
1	0	3	⁻2	5	⁻4	7
⁻2	⁻3	0	⁻5	2	⁻7	4
3	2	5	0	7	⁻2	9
⁻4	⁻5	⁻2	⁻7	0	⁻9	2
5	4	7	2	9	0	11
⁻6	⁻7	⁻4	⁻9	⁻2	⁻11	0

- *(When creating their own game)* What is your game going to be? How will it use negative numbers? Are your rules clear?

−1, 2, −3,
4, −5, 6
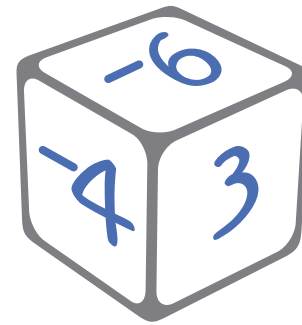
1, −2, 3,
−4, 5, −6

−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	10
−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	10

−20	−19	−18	−17	−16	−15	−14	−13	−12	−11	−10	−9	−8	−7	−6
−20	−19	−18	−17	−16	−15	−14	−13	−12	−11	−10	−9	−8	−7	−6

Glue here	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8

Glue here	9	10	11	12	13	14	15	16	17	18	19	20
	9	10	11	12	13	14	15	16	17	18	19	20